Can a Time-Varying Equilibrium Real Interest Rate Explain the Excess Sensitivity Puzzle?*

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Abstract

This paper analyses the strong responses of long-term interest rates to shocks that are difficult to explain with standard macroeconomic models. Augmenting the standard model to include a time-varying equilibrium real interest rate generates forward rates that exhibit considerable movement at long horizons in response to movements of the policy-controlled short rate. In terms of coefficients from regressions of long-rate changes on short-rate movements, incorporating a time-varying natural rate explains a significant fraction of the excess sensitivity puzzle.

Key words: Term structure, equilibrium real interest rate, unobserved components model
JEL classification: E43, E52, C51

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1 Introduction

Standard models of monetary policy assume that long-term interest rates should remain stable when shocks arrive in the economy. In contrast, observed long-term interest rates often exhibit considerable movement in the same direction as short-term interest rates, that is, the yield curve shifts in a parallel manner. This behaviour has been labelled the *excess sensitivity* and/or *excess volatility puzzle*. The excess volatility puzzle denotes the finding that the variance of long-term interest rates is far larger than standard models predict,[1] whereas the excess sensitivity puzzle concerns the large movements in long-term interest rates associated with changes of the policy-controlled short rate. Because we are primarily interested in explaining the prevalence of parallel shifts of forward rates and the yield curve, our focus here is on the excess sensitivity puzzle.

The excess sensitivity puzzle has been studied by Gürkaynak et al. (2003), Beechey (2004) and Ellingsen and Söderström (2005) amongst others. Gürkaynak et al. (2003) conclude that long-run inflation expectations are a crucial component of the puzzle and that the assumption of a time-invariant steady state is incapable of generating such behaviour. More specifically, they suggest that the private sector adjusts its expectations of the long-run inflation target in response to macroeconomic shocks in a manner that causes the observed movements of nominal long-term interest rates. Ellingsen and Söderström (2005) show that a model with private central bank information about future inflation generates moderate parallel shifts in the yield curve when the economy is hit by supply and demand shocks. They estimate that the ten-year interest rate rises on average by 25 basis points in response to an unexpected one percentage point increase in the policy-controlled short rate. Beechey (2004) shows that a non-stationary inflation target and adaptive learning about the target can also generate sensitivity of long-term interest rates. Overall, existing explanations focus on whether the central bank has a credible and known inflation target.

This paper investigates an alternative solution to the excess sensitivity puzzle, namely that it may be due to persistent effects of shocks to the equilibrium real interest rate. Models which are typically used to illustrate the excess sensitivity puzzle assume that the equilibrium interest rate is (i) constant over time and (ii) independent of structural shocks. In contrast, many dynamic stochastic general equilibrium (DSGE) models share the features that shocks to technology or consumers' rate of time preference cause persistent movements in the equilibrium real rate. As supporting evidence, several studies find that the equilibrium real rate can be modelled as non-stationary (Laubach and Williams,

[1] See e.g. Shiller (1979) for an early discussion.

2003) or near unit root process (Mésonnier and Renne, 2004), implying that shocks have permanent effects or at least highly persistent effects.[2]

Following the just cited contributions we use an unobserved components model estimated with the Kalman filter to extract a measure of the equilibrium real rate. We then show that a general equilibrium model augmented with the estimated process has the potential to explain the excess sensitivity puzzle. Relative to the case when the equilibrium real interest rate is assumed constant, forward-interest rates at long horizons respond more to short-rate movements when shocks have persistent effects on the equilibrium real rate. An alternative empirical formulation of the excess sensitivity puzzle is that long rates react by more than predicted to changes in the short rate. Subject to the monetary policy reaction function that we employ, are we able to show that the slope coefficients from regressions of changes in long-term interest rates on changes in short-term interest rates are about 10 basis points higher when the equilibrium real interest rate is time-varying than when it is constant.

One motivation for focusing on the equilibrium real interest rate rather than informational aspects such as inflation expectations is the finding by Pennacchi (1991) that real interest rates are more volatile than expected inflation. An implication of this finding is that variation in long-term interest rates is primarily due to shocks to the real interest rate. As such, it deserves attention as an explanation for the excess sensitivity puzzle. Other authors have reached the opposite conclusion, that is that nominal volatility is more important than real volatility and we do not deny that inflation expectations and/or a time-varying inflation target may also play a role. However, in this study attention will be confined to real explanations. We find that equilibrium real interest rates display sufficient variation and dependence on structural shocks to come close the observed co-movements of long-term interest rates and short-term interest rates.

To generate persistent effects on interest rates, either the stochastic process of the shock itself or the effects of the shock on other variables in the model need to be highly persistent. Gürkaynak et al. (2003) conclude that the degree of persistence of nominal shocks required to solve the puzzle is unreasonably high. Our contribution is to show that the effects of real shocks propagated through the equilibrium real rate is plausibly high enough to contribute to the model's predictions with observed data.

The paper is organised as follows. Section 2 presents a semi-structural general equilibrium model with a time-varying equilibrium real rate. In Section 3 we estimate this time-varying real rate. Section 4 analyses the implications for the excess sensitivity puzzle in two ways: first by examining the effect of shocks on long-horizon forward rates and

[2]Other contributions in the same vein are Andrés et al. (2004) and Manrique and Marqués (2004).

second through the coefficients of regressions of long-rate changes on short rate movements. Section 5 concludes.

2 A Stylised Model

We employ the stylised model estimated by Rudebusch and Svensson (1999) and Rudebusch (2002), used in Gürkaynak et al. (2003) and Ellingsen and Söderström (2005) for the analysis of excess sensitivity of long-term interest rates. As discussed in Rudebusch and Svensson (1999) and Rudebusch (2002), such a model can account for several broad characteristics of larger models commonly used by central banks. It also shares features of modern DSGE models, including both forward-looking and backward-looking terms in the Phillips curve and aggregate demand equation. However, the model used in this paper features richer dynamics than standard sticky-price models that are usually parsimonious in the lag structure but based on firmer microfoundations.

Let y_t denote the output gap measured as the percent deviation of output from its potential, π_t, the annualised quarterly inflation rate and $r_t \equiv i_t - E_{t-1}\overline{\pi}_{t+3}$ the real interest rate measured as the difference between the nominal interest rate and four-quarter ended inflation expectations. Inflation expectations are calculated as $E_{t-1}\overline{\pi}_{t+3} \equiv E_{t-1}\frac{1}{4}\sum_{j=0}^{3}\pi_{t+j}$. Aggregate demand and is affected by monetary policy via the real interest rate gap, the deviation of the real rate from its equilibrium level, r_t^*, and its backward-looking component is an AR(2) process,

$$y_t = \phi_y E_{t-1}y_{t+1} + \left(1 - \phi_y\right)\sum_{s=1}^{2}\alpha_{ys}y_{t-s} - \alpha_r(r_{t-1} - r_{t-1}^*) + \varepsilon_t^y. \quad (1)$$

The equilibrium real rate is defined as the real interest rate that has a neutral effect on demand, that is the real interest rate consistent with a zero output gap given that the economy is in equilibrium. ϕ_y governs the degree of forward-looking and ε_t^y is a zero mean *iid*-shock.

The Phillips curve allows for both backward-looking and forward-looking behaviour as well, where ϕ_π is the weight on expected inflation and $1 - \phi_\pi$ is the weight on lags one to four of inflation.

$$\pi_t = \phi_\pi E_{t-1}\overline{\pi}_{t+3} + (1 - \phi_\pi)\sum_{s=1}^{4}\beta_{\pi s}\pi_{t-s} + \beta_{y1}y_{t-1} + \varepsilon_t^\pi. \quad (2)$$

There is a continuing debate about the magnitude of the parameters ϕ_y and ϕ_π; Estrella and Fuhrer (2002) amongst others have argued that purely forward-looking frameworks are empirically implausible whereas Rudebusch (2002) and Clarida et al. (2000) argue that expectations about future inflation are an important component in hybrid models. The results also depend on the type of model and on the assumed policy regime. In estimated DSGE models with a simple Taylor rule chosen to describe monetary policy, it is often found that ϕ_π is relatively high, in the range of 0.7, whereas forward-looking in consumption is low.[3] However, Söderlind et al. (2005) estimate a model like ours assuming that the central bank implements a discretionary optimal monetary policy with the objective to minimise the weighted unconditional variances of inflation, the output gap and the change in the nominal interest rate. They find that inflation expectations play a minor role whereas expectations about the output gap are important. On the other hand, single equation estimations of equation (1) and (2) seem to confirm the results found in DSGE models.[4]

In Rudebusch's specification and other models used to analyse the excess sensitivity puzzle, the equilibrium real rate is assumed to be constant over time and unaffected by structural shocks, i.e. $r_t^* = r$ for all t. However, as mentioned in the introduction, there is abundant empirical evidence that the equilibrium real rate shifts over time, in particular in response to shocks hitting the economy. Therefore, we model the equilibrium real rate as an autoregressive progress that is driven by the demand disturbance ε_t^y and own disturbances, ε_t^*,

$$r_t^* = \rho_r r_{t-1}^* + \rho_e \varepsilon_t^y + \varepsilon_t^*. \tag{3}$$

Recalling that the aggregate demand equation is formulated in terms of the output gap, we show in Appendix A that this dynamic specification can be derived by assuming that two persistent processes affect the output gap: a persistent aggregate demand shock and, for example, a persistent productivity shock. As shown in Laubach and Williams (2003), a link between the equilibrium real rate and trend productivity can be derived from a stochastic growth model. The combination of two persistent processes leads generally to a more complex dynamic process but can be simplified to the expression in equation (3) by assuming that both processes share the same degree of persistence.[5] In addition, our structural model can be viewed as a stylised version of more elaborate models which account for an explicit link between the marginal product of capital and the real interest rate. Specifically in DSGE models that feature endogenous investment decisions, the equilibrium real interest rate depends on the capital stock, itself a persistent variable (Woodford, 2003). These features motivate our choice of equation (3). As is shown in Appendix A,

[3] See e.g. Smets and Wouters (2003) and Welz (2005).

[4] Galí and Gertler (1999) estimate ϕ_π around 0.8 and Fuhrer and Rudebusch (2004) find values for ϕ_y between 0 and 0.45.

[5] We assume the two shock processes to be AR(1); the sum of two AR(1)-processes that have equal persistence ρ is an AR(1)-process with the same persistence ρ.

the shock ε_t^* can be interpreted as capturing effects on the equilibrium real rate that do not have a contemporaneous effect on the output gap or inflation but act with a one-period delay. As the model is not fully structural, we label it for the remainder of the paper *equilibrium real rate shock*. The timing assumption is our central identifying criteria between the two shocks.

Allowing for a highly-persistent, time-varying equilibrium real rate in the aggregate demand relation will increase persistence in the nominal interest rate if the central bank responds to it. Hence, the model is closed by adding a Taylor rule which permits interest-rate smoothing, a common empirical finding; we will consider both $f_i = 0$ and $f_i \neq 0$ in

$$i_t = f_i i_{t-1} + (1 - f_i)(r_t^* + f_\pi \pi_t + f_y y_t) + \varepsilon_t^i. \tag{4}$$

Note that a constant equilibrium real rate (and inflation target) would appear as an intercept term in (4). We assume that the inflation target is constant and equal to zero, in contrast to Gürkaynak et al. (2003) and Ellingsen and Söderström (2005) who suggest a time-varying inflation target to explain the excess sensitivity puzzle. Most parameter values for the model are taken from Rudebusch (2002). Parameters pertaining to the time-varying equilibrium real interest rate need to be estimated in order to calibrate the model and analyse the response of the short-term and long-term interest rates to shocks.

3 Estimating a Time-Varying Equilibrium Real Rate

The extent to which a time-varying equilibrium real rate can explain the excess sensitivity puzzle depends on the size of the parameters ρ_r and ρ_e in equation (3) and the standard deviations of the shocks. If the equilibrium real rate is highly autocorrelated or close to a random walk, only small shocks are required to create large co-movements between long-term interest rates and short-term interest rates. In this section we estimate a time-varying equilibrium real rate using an unobserved components model.

3.1 Data

All data is measured at a quarterly frequency and obtained from the Federal Reserve Bank of St. Louis database (FRED) covering the period 1959Q1-2004Q3. For the level of potential output (y^{pot}) we use the series provided by the Congressional Budget Office (CBO), where output (y_t) is measured as chained real GDP in billions of year 2000 U.S.

dollars. We construct the output gap as $\widetilde{y}_t = 100\log(y_t/y_t^{pot})$. The nominal interest rate is measured as the quarterly average of the monthly Federal Funds rate and the real interest rate is defined as the difference between the nominal interest rate and the year-on-year inflation rate measured by the percentage change in the GDP chained price index.

3.2 Empirical Specification

We do not estimate the complete model from the previous section but rather focus on the aggregate demand equation and the dynamic specification for the equilibrium real interest rate. However, the empirical specification is similar to those of Laubach and Williams (2003) and Mésonnier and Renne (2004). These authors estimated the equilibrium real rate and potential output jointly. In our approach potential output is treated as an observable variable and the empirical model is formulated as follows:[6]

$$y_t = \alpha_{y1} y_{t-1} + \alpha_{y2} y_{t-2} - \alpha_r \widetilde{r}_{t-1} + \varepsilon_t^y \quad (5)$$

$$r_t^* = \rho_r r_{t-1}^* + \rho_e \varepsilon_t^y + \varepsilon_t^* \quad (6)$$

$$\widetilde{r}_t = d_1 \widetilde{r}_{t-1} + d_2 \widetilde{r}_{t-2} + \varepsilon_t^r \quad (7)$$

$$\widetilde{r}_t = r_t - r_t^* \quad (8)$$

The first equation is the empirical counterpart of our aggregate demand equation (1), that is we do not attempt to estimate the forward-looking component of output, as this would be an additional unobservable variable. The second equation describes the dynamics of the equilibrium real interest rate. We postulate a stationary AR(2)-process for the real-rate gap in equation (7), assuming that it follows similar dynamics to the (backward component of the) output gap.[7] Note that our model nests stationary and nonstationary dynamics for the equilibrium real rate, where nonstationarity implies that r_t and r_t^* are cointegrated as long as the real rate gap is stationary, which we restrict it to be.

In addition to treating potential output as observable variable, differs our specification from those of Laubach and Williams (2003) and Mésonnier and Renne (2004) with respect to equations (6) and (7). The former authors postulate that r_t^* is nonstationary because in their model the equilibrium real rate depends on the trend growth rate of potential output, which is itself driven by a random walk. Mésonnier and Renne (2004) postulate the same joint link between the trend growth rate and potential output and the equilibrium real rate but assume that this growth rate is stationary. In our model we do not explicitly relate

[6] See Clark and Kozicki (2004) for a comparison of models that jointly estimate the equilibrium real rate of interest and potential output with models that only estimate the equilibrium real rate of interest given the CBO-measure of potential output.

[7] A similar approach has earlier been used to estimate potential output, for example by Clark (1987).

the equilibrium real rate to the trend growth rate of output but only approximate it to be autocorrelated and focus on its dependence on shocks. As stated before, demand shocks have contemporaneous effects on the output gap and the equilibrium real rate whereas real rate shocks influence the output gap with one-period delay. In the absence of shocks, equation (5) implies that in the long run a zero output gap corresponds with a zero interest rate gap, in line with our definition of the equilibrium real rate.

It would obviously be desirable to estimate the entire model rather than focusing on the time-varying equilibrium real rate. A long sample period is essential for capturing movements in the equilibrium real rate but we have not succeeded in obtaining a satisfactory empirical specification of the whole theoretical model with constant parameters over such a long sample. In particular, the monetary policy reaction function appears unstable, possibly because the estimated time interval spans several different monetary policy regimes. Against this background it is not clear how to treat for instance the inflation target. Hence, we do not take a particular stance on monetary policy here but simply postulate in equation (7) that the Federal Reserve has conducted monetary policy in a way that the deviation of the real interest rate from its equilibrium is stationary over the estimation period. This is a general assumption that encompasses a dynamic Taylor rule of the type in our theoretical model.

All shocks in the model are assumed independent of each other, implying a diagonal variance-covariance matrix of the transition equation

$$\Sigma = \begin{bmatrix} \sigma_y^2 & & \\ & \sigma_{\tilde{r}}^2 & \\ & & \sigma_{r*}^2 \end{bmatrix}.$$

The model is written in state-space form (see Appendix B for a detailed representation) and the value of the likelihood function calculated with the Kalman filter. The likelihood function is maximised by standard procedures and the negative inverse Hessian is computed in order to find the standard errors of the estimates.[8] In the following section the estimation results are discussed.

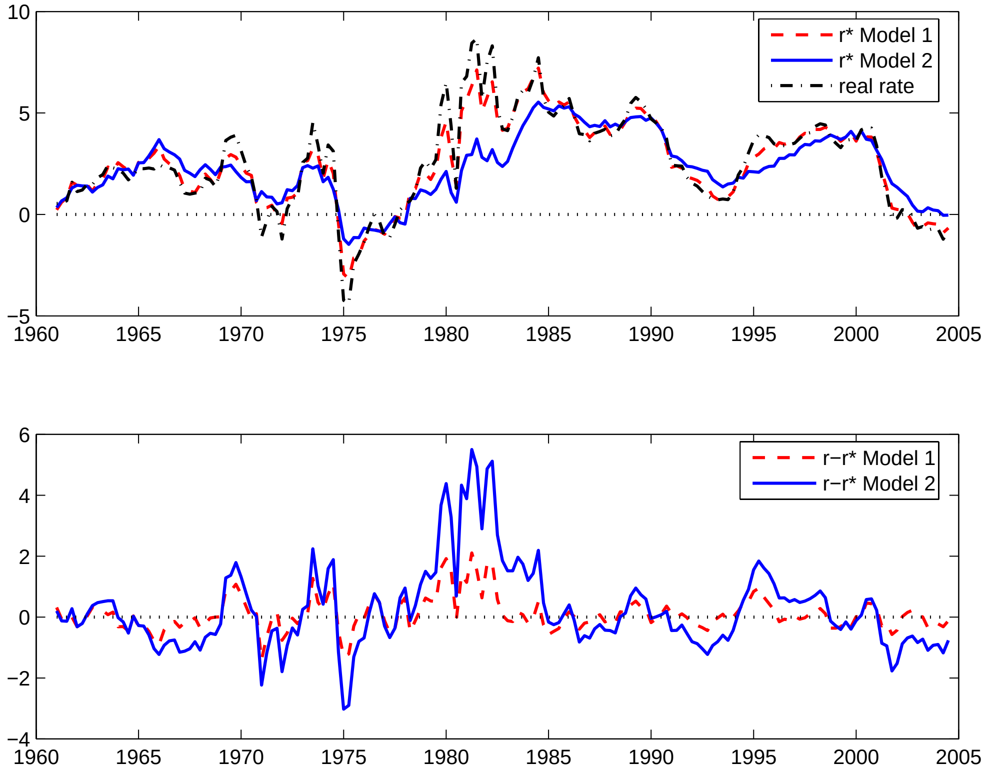
3.3 Results

For comparison, two models are estimated. Model 1 estimates all parameters freely whereas Model 2 fixes the standard deviation of the equilibrium real rate to $\sigma_{r*} = 0.322$;

[8]Since we assume that the equilibrium real rate is stationary, a proper prior distribution of the state vector can be found. The initial covariance matrix is computed by $vec(P_{1|0}) = (I - T \otimes T)^{-1} vec(RQR')$, the means of the real rate gap and the output gap are set to zero and the mean of r_t^* is set equal to the mean of the real interest rate over the sample period (see Appendix B for details of the state space model and the notation). For calculation of the log-likelihood we use Paul Söderlind's Kalman filter Gauss code adapted to Matlab.

this value is estimated by Clark and Kozicki (2004) assuming that r_t^* follows a random walk. The smaller standard deviation leads to a smoother estimate of the equilibrium real rate relative to Model 1. The equilibrium interest rate series is intrinsically difficult to measure; for instance, Orphanides and Williams (2002) compare six alternative methods of measuring the equilibrium real rate and find considerable differences depending on the method used. With regard to the smoothness of the equilibrium real rate, it is not immediately obvious that it should be smoother than the actual real interest rate. For example, Smets and Wouters (2003) find that the equilibrium real rate, defined as the real interest rate under flexible prices and absent nominal shocks, varies *more* than the real interest rate.

Figure 1: Estimated Equilibrium Real Rate



Notes: Top panel: Estimated equilibrium real interest rates from Model 1 (dashed line) and Model 2 (solid line) and the real interest rate (dash-dotted line).
Bottom panel: Real interest rate gaps from Model 1 (dashed line) and Model 2 (solid line).

Figure 1 graphs the equilibrium real rates generated from the two estimated models. The top panel plots the estimates from the one-sided Kalman filter together with the observed real interest rate. The bottom panel presents the interest rate gaps from the two models. The estimated interest rate series from Model 1 tracks the real rate closely but the lower panel reveals sizeable variation in the real rate gap up to 2 percentage points.

As expected, Model 2 produces a flatter estimate of the equilibrium real interest rate.

Table 1: Estimation Results

Parameter	Model 1	Model 2
α_{y1}	1.061 (11.26)	1.111 (14.12)
α_{y2}	−0.118 (−1.30)	−0.161 (−2.00)
α_r	0.366 (2.47)	0.168 (2.46)
d_1	0.965 (5.65)	0.923 (8.89)
d_2	−0.277 (−2.03)	−0.116 (−1.14)
ρ_r	0.977 (54.25)	0.987 (89.91)
ρ_e	0.257 (2.43)	0.316 (2.82)
σ_y	0.731 (14.26)	0.786 (18.16)
$\sigma_{\tilde{r}}$	0.595 (4.01)	0.827 (15.86)
σ_{r*}	0.662 (4.60)	0.322 (–)
Log-likelihood	-440.58	-444.69
AIC / BIC	-4.978 / -4.796	-5.037 / -4.873

Notes: In Model 2, σ_{r*} is fixed to 0.322.
AIC (BIC) = Akaike (Bayesian) information criterion

The main purpose of this study is not to find the most accurate measure of the equilibrium real rate but to document that it is time-variant, highly persistent and affected by real shocks. Both of our estimated models produce an estimate of the equilibrium real rate that fulfills these criteria. The coefficient estimates for the two models are shown in Table 1. The equilibrium real rate appears to be highly persistent with ρ_r estimated as 0.98 and 0.99 respectively. In Appendix C we show plots of the conditional likelihoods in an interval around the optimum for each parameter. These plots confirm that ρ_r should be below 1 and thus the equilibrium real rate should be stationary. The other coefficient estimates are similar across the two models, with the exception of α_r (the sensitivity of the output gap with respect to the real rate gap) which has a smaller value in Model 2. The lower value is similar to that estimated in models assuming a constant equilibrium real interest rate (Rudebusch, 2002) but also falls closer to the range of estimates rates found by Laubach and Williams (2003).[9]

[9]They estimate α_r between 0.088 and 0.122. Appendix D shows graphs of the relationship between the lagged interest rate gap and the output gap.

4 The Puzzle Explained?

We employ two types of output from the model to assess whether a time-varying equilibrium real interest rate can resolve the excess sensitivity puzzle. We begin by deriving the impulse response function of the short-term interest rate to various shocks. The desired empirical property is that the model is capable of generating significant movements at 20 to 40 quarter horizons. Next we generate time series data of the term structure from the model and regress changes in the artificial long-term interest rates on changes in the short-term rate. If the long-term (ten-year) interest rate attracts a coefficient of 0.2 or more, we conclude that our approach has the potential to explain the excess sensitivity puzzle. In both exercises, we compare results from the model with the time-varying equilibrium real rate to the model where the rate is constant.

The model is calibrated using values from Rudebusch (2002) and our estimated parameters for the time-varying equilibrium real rate.[10] Because the equilibrium real rate is constant in the standard Rudebusch specification, he does not provide parameter values for ρ_r and ρ_e. All parameter values are shown in Table 2. For the empirical difficulties discussed before, we also choose the standard parameter values for the Taylor rule suggested by Taylor (1993) and perform a sensitivity analysis with respect to interest-rate smoothing and the output gap coefficient.

Two main caveats to the analysis need to be stated. First, we have made the strong assumption that the central bank has full information about the equilibrium real interest rate in the same period that it sets the nominal short-term interest rate. Assuming instead that the central bank observes the equilibrium real rate imperfectly would serve to strengthen our results but would conflict with the assumption of full information about the output gap. There is a rich empirical and theoretical literature that discusses the implications for monetary policy that arise from imperfect information about the stance of the economy in general and about natural rates in particular. Orphanides (2001) shows, for instance, that the use of real-time data instead of ex-post revised data changes the recommendations for monetary policy considerably. Furthermore, Orphanides and Williams (2002) discuss the performance of Taylor-type monetary policy rules in the presence of imperfect knowledge of the natural rate of unemployment and the equilibrium real rate of interest. In our empirical model we take account of imperfect knowledge about the equilibrium real rate of interest only to the extent that we employ a one-sided Kalman filter rather than the smoothed estimates.

[10] Because this model cannot be solved analytically we use algorithms provided in the Dynare-package for Matlab to obtain numerical simulated responses of the nominal interest rate i_t to shocks for specific sets of parameter values.

Table 2: Model Calibration

Aggregate Supply	Aggregate Demand	Monetary Policy	Equilibrium real Interest rate
$\phi_\pi = 0.29$	$\alpha_{y1} = 1.15$	$f_\pi = 1.50$	$\rho_r = 0.987$
$\beta_{\pi 1} = 0.67$	$\alpha_{y2} = -0.27$	$f_y = 0.50$	$\rho_e = 0.316$
$\beta_{\pi 2} = -0.14$	$\alpha_r = 0.09$	$f_i = 0\ (0.50)$	$\sigma_{r*} = 0.322$
$\beta_{\pi 3} = 0.40$	$\sigma(\varepsilon^y) = 0.833$	$\sigma(\varepsilon^i) = 1$	
$\beta_{\pi 4} = 0.07$			
$\beta_y = 0.13$			
$\sigma(\varepsilon^\pi) = 1.012$			

Sources: Rudebusch (2002) (aggregate demand and supply), Taylor (1993) (monetary policy), own estimates (equilibrium real interest rate).

The second caveat is that we have assumed the coefficients in the Taylor rule are unchanged when the time-varying equilibrium real interest rate is introduced. Given that a long estimation horizon is needed to estimate the equilibrium real interest rate, it is difficult to estimate a single time-invariant monetary policy rule. Hence we have chosen to adopt the parameters suggested by Taylor (1993).

4.1 Impulse Responses of Nominal Interest Rates

Constant Equilibrium Real Rate

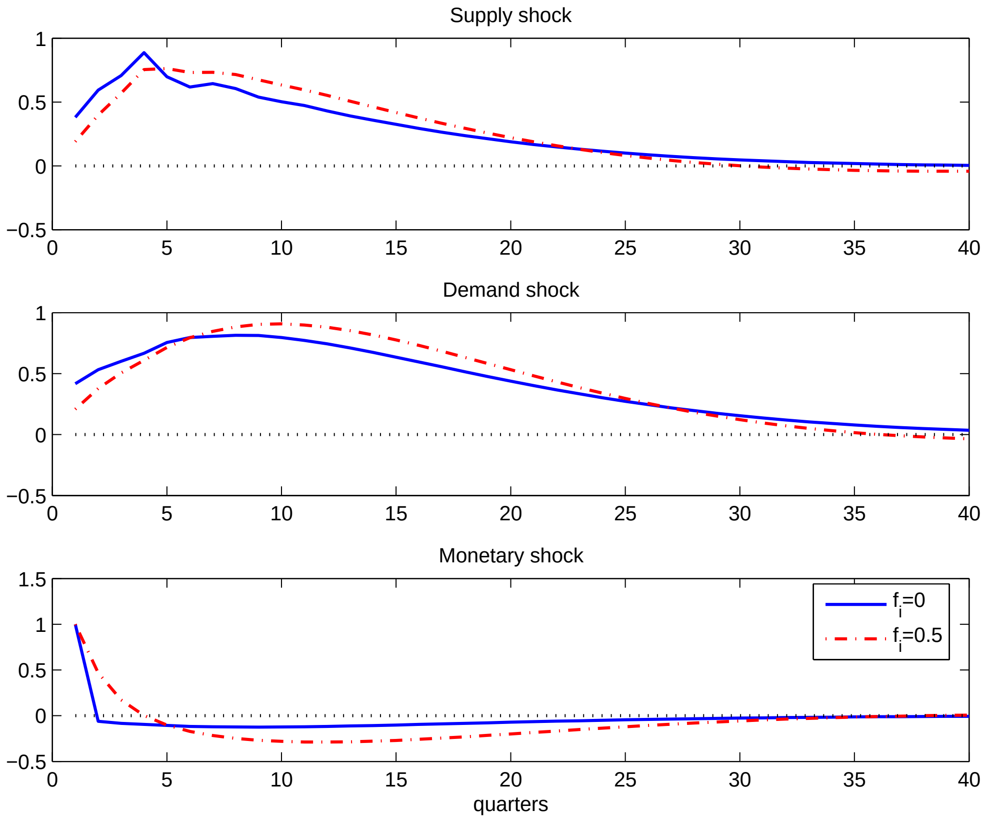
Figure 2 presents impulse responses of the future short-term nominal interest rate one to 40 quarters into the future for the model with a constant equilibrium real rate. These are the predicted forward-rate responses to shocks to aggregate supply, aggregate demand and the nominal short-term interest rate when the equilibrium real rate is constant.

The impulse responses reveal that nominal interest rates return quickly to zero. For instance, following a demand shock, forward rates move by only 3.5 basis points after 40 quarters in the version with no interest-rate smoothing (static Taylor rule). This implies that forward interest rates respond very little at long horizons to short rate movements and is at odds with the empirical facts. For instance Gürkaynak et al. (2003) show that forward rates at the 10-year horizon move significantly in response to news releases of a broad set of macroeconomic indicators.

Time-Varying Equilibrium Real Rate

We now turn to the case of a persistent time-varying equilibrium real rate and investigate the impulse response of the nominal short rate when the equilibrium real rate is hit by a demand shock or an equilibrium real rate shock.

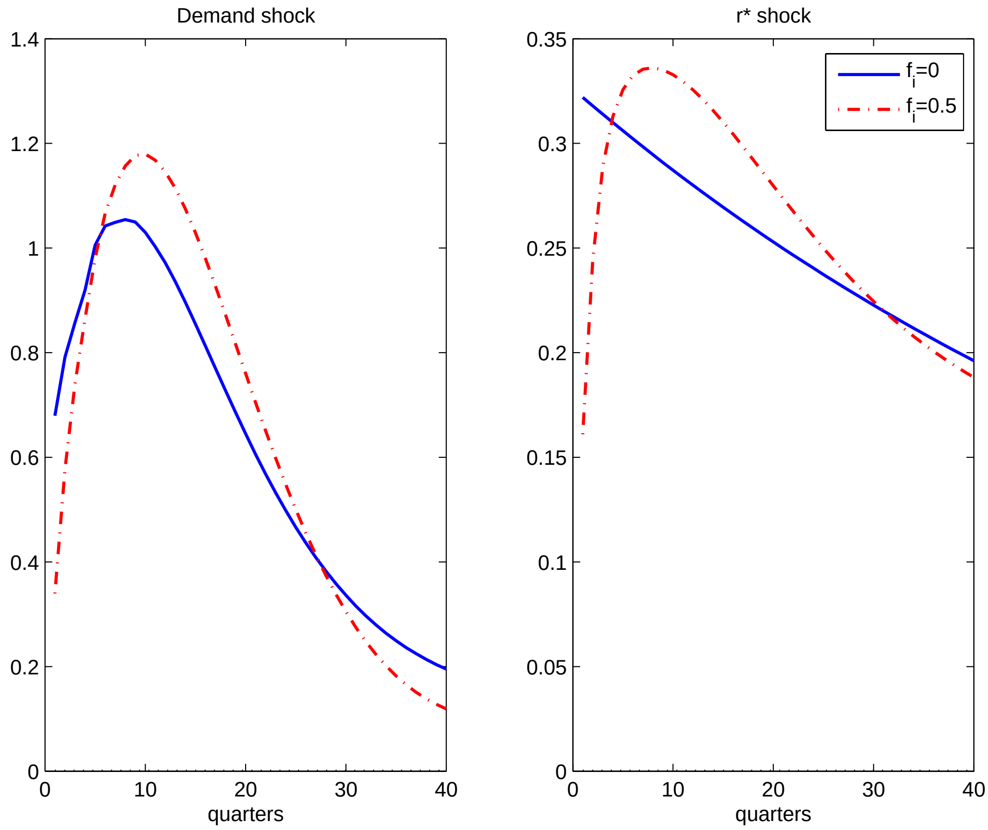
Figure 2: Model With Constant Equilibrium Real Rate



Notes: Impulse responses of the nominal short-term rate in the model with constant equilibrium real rate, with static (solid line, $f_i = 0$) and dynamic (dash-dotted line, $f_i = 0.50$) Taylor rule.

Figure 3 shows the response to an aggregate demand shock in the left panel and the response to an equilibrium real rate shock in the right panel. As in Figure 2, the abscissa denote the time horizon of the short-term interest rate response as well as the forward-rate horizon. Because the equilibrium real rate shock is a 'new' source of fluctuation, only the response to the demand shock ε_t^y can be compared to Figure 2. Given the assumed parametrisation of the model, there is now a considerable effect on forward interest rates at the ten-year horizon from a demand shock (left panel): in absolute terms the forward rate moves 12 basis into the same direction as the short rate when the Taylor rule is dynamic. This corresponds to a shift of 29 basis points when the short rate moves 100 basis points. Similarly, if the Taylor rule is static, forward rates move 20 basis points in absolute terms, or 35 basis points relative to a 100 basis point short-rate rise. The effects are stronger in response to an equilibrium real rate shock (right panel), namely 19 basis points, or 61 points when the short rate moves 100 basis points, in the presence of interest-rate smoothing. In the case with no smoothing, the forward rate responds by more than the short rate, namely by 117 basis points relative to a 100 basis point shift in the

Figure 3: Model with Time-Varying Equilibrium Real Rate



Notes: Responses of the nominal short-term rate to a demand shock (left panel) and an equilibrium real rate shock (right panel), with static (solid line $f_i = 0$) and dynamic (dash-dotted line $f_i = 0.50$) Taylor rule.

policy-controlled rate. The effects in the no-smoothing case appear to be longer-lasting. With interest-rate smoothing the economy experiences larger swings because the central bank first responds relatively little to the shocks but subsequently increases the nominal rate by more than in the no-smoothing case. As a result, the inflation and output gaps are closed more quickly. Beyond 40 quarters, when the inflation and output gaps have nearly returned to equilibrium, the dynamics of the nominal interest rate are dominated by the high persistence of the equilibrium real rate. From this horizon onward, the two cases produce quantitatively similar results.

4.2 Regression Evidence from Interest Rate Changes

The original formulation of the excess sensitivity puzzle concerns the large *average* response of long-term interest rates to changes in the policy-controlled short rate. Hence, in this section we investigate the overall impact of a time-varying equilibrium real rate on long-rate fluctuations relative to short rate movements. Several authors have distinguished between expected and unexpected changes in monetary policy, based on the reasoning that expected events have already been factored into long-term interest rates. Ellingsen and Söderström (2005) report that an unexpected one percentage point rise in the Federal Funds rate is associated with a 25 basis point rise in ten-year yields.

From the model we generate time-series data (10 000 observations) on the policy-controlled short rate and the ten-year interest rate calculated as

$$i_t^n = \frac{1}{n} \sum_{s=0}^{n-1} E_t\{i_{t+s}\}. \tag{9}$$

First differences of the long-term rate are then regressed on the short-term rate. Because we construct the long-term rate according to the expectations hypothesis, only unexpected changes in the short-term rate affect longer yields. Hence it is not necessary to distinguish between expected and unexpected monetary policy changes in this analysis. Four cases are examined to assess the effects of a time-varying equilibrium real rate and separate them from the effects of interest-rate smoothing. Specifically, we consider cases with and without interest-rate smoothing, a constant and time-varying equilibrium real interest rate and combinations thereof.

Table 3: Regression Results

	Equilibrium Real Rate	
Taylor Rule	time-varying	constant
static	0.20	0.10
dynamic	0.13	0.04

Note: Slope coefficients from regressing changes in long-term interest rates on changes in short-term interest rates $\Delta i_t^{40} = \alpha + \beta \Delta i_t^1 + \epsilon_t$

Table 3 reports the results from regressing changes in long-term interest rates on changes in the policy-controlled short rate. As can be seen from the table, relative to the model with constant equilibrium real rate the β-coefficient is estimated to be about 10 basis points larger when the equilibrium rate is time-varying at the 10-year horizon. Inclusion of a time-varying equilibrium real interest rate explains about 25 percent of the excess sensitivity puzzle.

4.3 Sensitivity Analysis

In addition to distinguishing between a dynamic and static Taylor rule, we perform a small-scale sensitivity analysis to the results in Table 3 with respect to the output gap coefficient. In crude single equation estimations of the Taylor rule (4) that assume a constant equilibrium real rate, this coefficient turns out to be rather unstable over the sample period, ranging from values close to zero to values above 1.The inflation coefficient is more stable around 1.5 by comparison. We solve the theoretical model for the additional cases $f_y \in \{0, 1\}$ keeping all other coefficients fixed and simulate again data from the model. While $f_y = 0$ (= 1) shifts the β−coefficient down (up), the 10 basis point difference between the model with constant and time-varying equilibrium rate is preserved.

5 Conclusions

This paper explores one explanation for the excess sensitivity of long-term interest rates, namely the covariance between the equilibrium real rate and the policy-controlled short rate when both react to the same shocks. Models of the monetary transmission mechanism used to analyse the excess sensitivity puzzle assume that the equilibrium real rate is constant over time and unrelated to structural shocks. However, empirical evidence has shown that equilibrium real rates do vary considerably over time and this fact is embedded in state-of-the-art DSGE models. We incorporate this feature into a familiar semi-structural general equilibrium model frequently employed in the monetary policy literature.

We estimate the equilibrium real rate for the U.S. over the last 45 years using an unobserved components model which yields parameter values for a dynamic equilibrium real rate equation. The results suggest that the equilibrium real rate displays time variation and dependence on structural shocks, and confirm the findings of previous studies that the equilibrium real rate is highly persistent. Specifically, it is driven by demand shocks and own shocks, where the latter could be interpreted as a proxy for shocks to productivity.

Given the estimated parameters we show that a time-varying equilibrium real rate influenced by structural shocks can account for greater co-movement of short-term and long-term interest rates than a constant real rate benchmark. Slope coefficients from regressing changes in long-term rates on changes in short-term rates are approximately 10 basis point higher at the 10-year horizon when the equilibrium real rate is allowed to vary than when it is assumed to be constant. Assuming that the central bank responds to the equilibrium real rate, shocks have long lasting effects on forward rates and thereby onto long-term interest rates.

The approach we take focuses on the real side of the economy and we are unable to address nominal shocks. Thus our claim that a time-varying equilibrium real interest rate may explain the sensitivity puzzle does not exclude other explanations, such as the informational asymmetries and imperfect knowledge discussed by Gürkaynak et al. (2003) and Ellingsen and Söderström (2005). However, allowing real shocks to affect the equilibrium real interest rate creates an additional mechanism through which the effects of shocks become more persistent. Developing models that incorporate both real and nominal approaches would be valuable as they would allow conclusions to be drawn about the explanatory power of each approach.

References

Andrés, J., López-Salido, J. D., and Nelson, E. (2004). Money and the natural rate of interest: Structural estimates for the uk, the us and the euro area. CEPR Discussion Paper Series 4337, Centre for Economic Policy Research.

Beechey, M. (2004). Excess sensitivity and volatility of long interest rates: The role of limited information in bond markets. Sveriges Riksbank Working Paper No 173, Sveriges Riksbank.

Clarida, R., Galí, J., and Gertler, M. (2000). Monetary policy rules and macroeconomic stability: Evidence and some theory. *Quarterly Journal of Economics*, 115:147–180.

Clark, P. K. (1987). The cyclical component of U.S. economic activity. *Quarterly Journal of Economics*, 102:797–814.

Clark, T. E. and Kozicki, S. (2004). Estimating equilibrium real interest rates in real-time. Deutsche Bundesbank Discussion Paper Series No 32/2004.

Ellingsen, T. and Söderström, U. (2005). Why are long rates sensitive to monetary policy? Working Paper, Bocconi University.

Estrella, A. and Fuhrer, J. C. (2002). Dynamic inconsistencies: counterfactual implications of a class of rational-expectations models. *American Economic Review*, 92(4):1013–1028.

Fuhrer, J. C. and Rudebusch, G. D. (2004). Estimating the Euler equation for output. *Journal of Monetary Economics*, 51(6):1133–1153.

Galí, J. and Gertler, M. (1999). Inflation dynamics: a structural econometric analysis. *Journal of Monetary Economics*, 44(2):195–222.

Gürkaynak, R. S., Sack, B., and Swanson, E. T. (2003). The excess sensitivity of long-term interest rates to economic news: evidence and implications for macroeconomic models. Federal Reserve Board: Finance and Economics Discussion Series 2003-50.

Laubach, T. and Williams, J. C. (2003). Measuring the natural rate of interest. *Review of Economics and Statistics*, 85(4):1063–1070.

Manrique, M. and Marqués, J. M. (2004). An empirical approximation of the natural rate of interest and potential growth. Documentos de Trabajo No. 0416, Banco de Espana, Madrid.

Mésonnier, J.-S. and Renne, J.-P. (2004). A time-varying 'natural' rate of interest for the Euro area. Working Paper.

Orphanides, A. (2001). Monetary policy rules based on real-time data. *American Economic Review*, 91(4):964–985.

Orphanides, A. and Williams, J. C. (2002). Robust monetary policy rules with unknown natural rates. *Brookings Papers on Economic Activity*, 2002:2:63–145.

Pennacchi, G. G. (1991). Identifying the dynamics of real interest rates and inflation : evidence using survey data. *The Review of Financial Studies*, 4(1):53–86.

Rudebusch, G. D. (2002). Assessing nominal income rules for monetary policy with model and data uncertainty. *Economic Journal*, 112:402–432.

Rudebusch, G. D. and Svensson, L. E. O. (1999). Policy rules for inflation targeting. In Taylor, J. B., editor, *Monetary Policy Rules*. Chicago University Press, Chicago.

Shiller, R. J. (1979). The volatility of long-term interest rates and expectations models of the term structure. *Journal of Political Economy*, 87(6):1190–1219.

Smets, F. and Wouters, R. (2003). An estimated dynamic stochastic general equilibrium model of the Euro area. *Journal of the European Economic Association*, 1(5):1123–1175.

Söderlind, P., Söderström, U., and Vredin, A. (2005). New-Keynesian models and monetary policy: A reexamination of the stylized facts. *Scandinavian Journal of Economics (forthcoming)*.

Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39:195–214.

Welz, P. (2005). Assessing predetermined expectations in the standard sticky price model: A Bayesian approach. Manuscript, Uppsala University.

Woodford, M. (2003). *Interest and Prices: Foundations of a theory of monetary policy*. Princeton University Press, Princeton, New Jersey.

Appendices

A Derivation of the Equilibrium Real Rate

This appendix shows how a specification of a time-varying equilibrium real interest rate such as equation (3) could be derived. Assume that there are two persistent shock processes influencing the aggregate demand relation; a 'demand' shock (e_t) and a 'productivity' or 'equilibrium real rate' shock (a_t), respectively and suppose that their dynamics can be modelled by stationary AR(1)-processes:

$$\begin{aligned} e_t &= \widetilde{\rho}_e e_{t-1} + \varepsilon_t^y && \text{(A1)} \\ a_t &= \rho_a a_{t-1} + \varepsilon_t^a. && \text{(A2)} \end{aligned}$$

Next assume that the demand shock process has a contemporaneous effect and the equilibrium real rate shock process has a one-period delayed effect on the output gap, i.e.

$$y_t = \phi_y E_{t-1} y_{t+1} + \left(1 - \phi_y\right)\left(b_{y1} y_{t-1} + b_{y2} y_{t-2}\right) - b_r r_{t-1} + e_t + a_{t-1} \qquad \text{(A3)}$$

so that

$$y_t = \phi_y E_{t-1} y_{t+1} + \left(1 - \phi_y\right)\left(b_{y1} y_{t-1} + b_{y2} y_{t-2}\right) - b_r r_{t-1} + \widetilde{\rho}_e e_{t-1} + a_{t-1} + \varepsilon_t^y. \qquad \text{(A3')}$$

Define the equilibrium real rate r_t^* as the real interest rate that implies $E_{t-1}\{y_t\} = 0$ given $y_{t-s} = 0$ for all $s > 0$. Then

$$\begin{aligned} 0 &= E_{t-1}\{y_t\} = -b_r r_{t-1}^* + \widetilde{\rho}_e e_{t-1} + a_{t-1} && \text{(A4)} \\ r_t^* &= \frac{1}{b_r}(\widetilde{\rho}_e e_t + a_t). && \text{(A4')} \end{aligned}$$

With $\widetilde{\rho}_e \neq \rho_a$, r_t^* follows an ARMA(2,1) process. Both processes are likely to be highly persistent and as an approximation we set $\rho_r \equiv \widetilde{\rho}_e = \rho_a$. Then equation (3) can be derived as follows

$$\begin{aligned} r_t^* &= \frac{1}{b_r}\left(\widetilde{\rho}_e^2 e_{t-1} + \rho_a a_{t-1} + \widetilde{\rho}_e \varepsilon_t^y + \varepsilon_t^a\right) && \text{(A5)} \\ &= \rho_r r_{t-1}^* + \frac{\widetilde{\rho}_e}{b_r}\varepsilon_t^y + \frac{1}{b_r}\varepsilon_t^a \\ &= \rho_r r_{t-1}^* + \rho_e \varepsilon_t^y + \varepsilon_t^* \end{aligned}$$

where $\rho_e \equiv \frac{\widetilde{\rho}_e}{b_r}$ and $\varepsilon_t^* \equiv \frac{\varepsilon_t^a}{b_r}$.

Finally, combining equation (A3') and (A4') yields aggregate demand with time-varying equilibrium real interest rate as in equation (1):

$$\begin{aligned} y_t &= \phi_y E_{t-1} y_{t+1} + \left(1-\phi_y\right)\left(b_{y1} y_{t-1} + b_{y2} y_{t-2}\right) - b_r r_{t-1} + \widetilde{\rho}_e e_{t-1} + a_{t-1} + \varepsilon_t^y \qquad \text{(A6)} \\ &= \phi_y E_{t-1} y_{t+1} + \left(1-\phi_y\right)\left(b_{y1} y_{t-1} + b_{y2} y_{t-2}\right) - b_r \left[r_{t-1} - \frac{1}{b_r}\left(\widetilde{\rho}_e e_{t-1} + a_{t-1}\right)\right] + \varepsilon_t^y \\ &= \phi_y E_{t-1} y_{t+1} + \left(1-\phi_y\right)\left(b_{y1} y_{t-1} + b_{y2} y_{t-2}\right) - b_r \left(r_{t-1} - r_{t-1}^*\right) + \varepsilon_t^y. \end{aligned}$$

B State Space Representation

Our state space model has the following general representation:

$$\begin{aligned} Y_t &= Z\xi_t && \text{(Measurement equations)} \\ \xi_t &= T\xi_{t-1} + R\varepsilon_t && \text{(Transition equations)} \end{aligned}$$

where $E(\varepsilon_t) = 0, E(\varepsilon_t \varepsilon_s) = 0$ for all $t \neq s$ and $E(\varepsilon_t \varepsilon_t') = Q$. The state vector has the following elements, $\xi_t = (\widetilde{y}_t, \widetilde{y}_{t-1}, \widetilde{r}_t, \widetilde{r}_{t-1}, r_t^*)'$ and the residual vector is comprised of $\varepsilon_t = \left(\varepsilon_t^y, \varepsilon_t^{\widetilde{r}}, \varepsilon_t^{r*}\right)'$.

Measurement equations

$\widetilde{y}_t = \widetilde{y}_t,$

$r_t = \widetilde{r}_t + r_t^*$

Transition equations

$\widetilde{y}_t = \alpha_{y1}\widetilde{y}_{t-1} + \alpha_{y2}\widetilde{y}_{t-2} - \alpha_r \widetilde{r}_{t-1} + \varepsilon_t^y$

$\widetilde{r}_t = d_1 \widetilde{r}_{t-1} + d_2 \widetilde{r}_{t-2} + \varepsilon_t^{\widetilde{r}}$

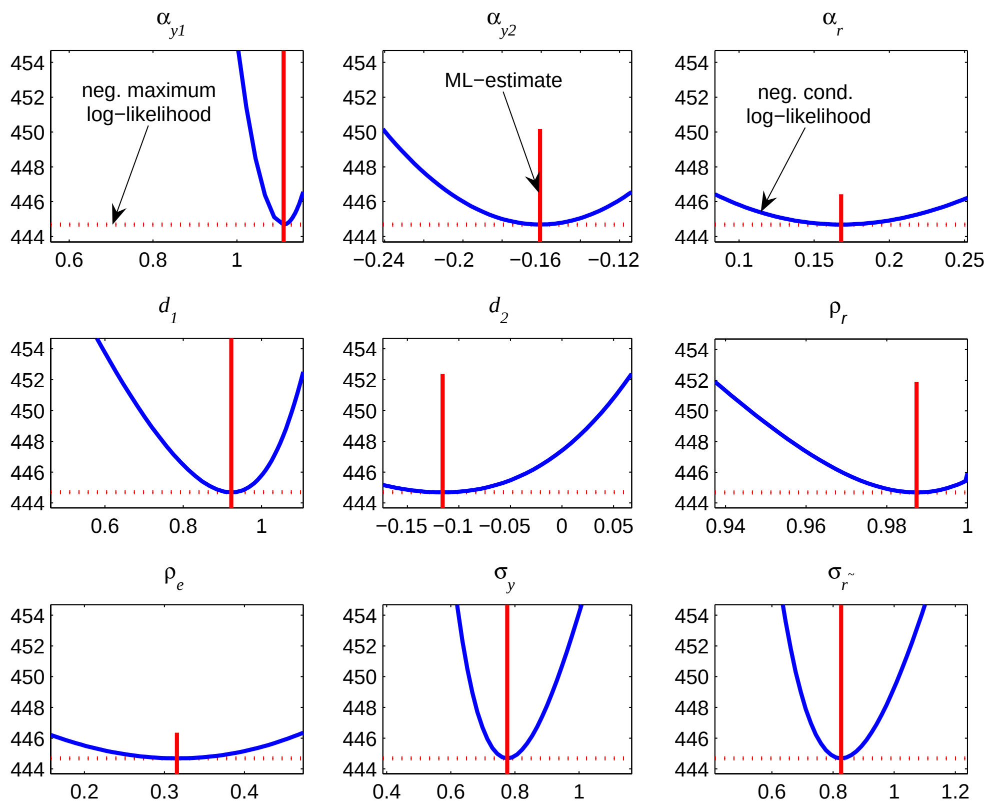
$r_t^* = \rho_r r_{t-1}^* + \rho_e \varepsilon_t^y + \varepsilon_t^{r*}$

$$Z = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}, \; T = \begin{bmatrix} \alpha_{y1} & \alpha_{y2} & -\alpha_r & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & d_1 & d_2 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_r \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ \rho_e & 0 & 1 \end{bmatrix} \qquad Q = \begin{bmatrix} \sigma_y^2 & & \\ & \sigma_{\widetilde{r}}^2 & \\ & & \sigma_{r*}^2 \end{bmatrix}$$

C Conditional Likelihoods

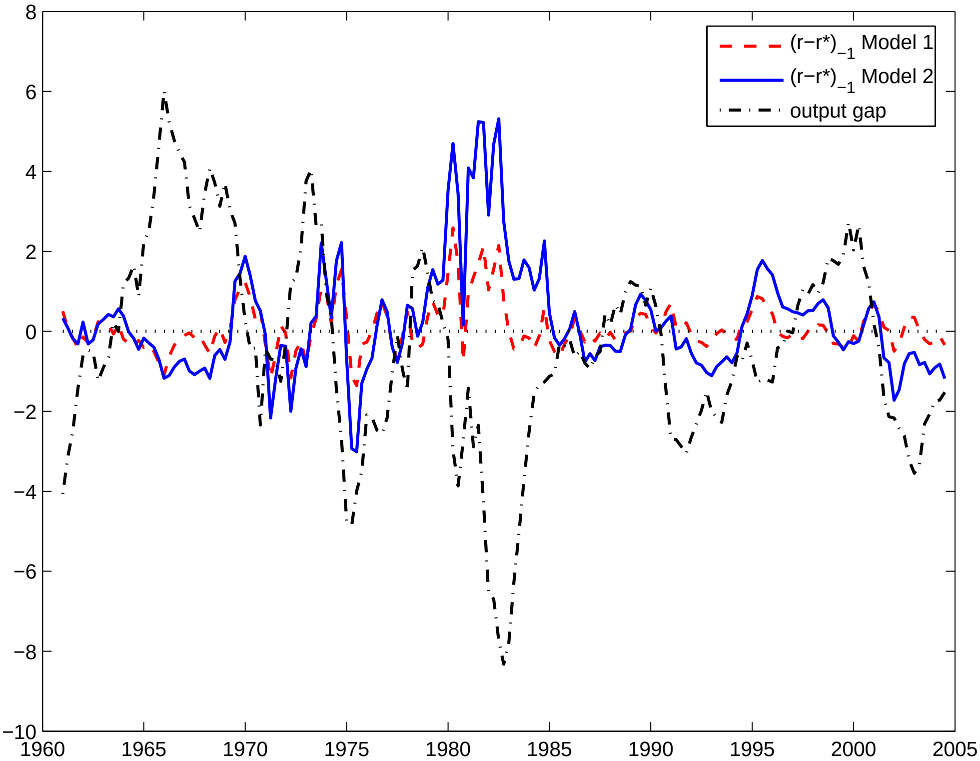
Figure 4: Optimisation Diagnostic



Notes: Negative conditional log-likelihoods around the optimum for Model 2. The plots are calculated with a diffuse prior on the initial state vector in order to allow the likelihood computation for $\rho_r = 1$ as well.

D Output Gap and Real Rate Gap

Figure 5: Output Gap and Real Rate Gap



Notes: One-period lagged real interest rate gap from Model 1 (dashed line) and Model 2 (solid line) and the output gap (dash-dotted line).